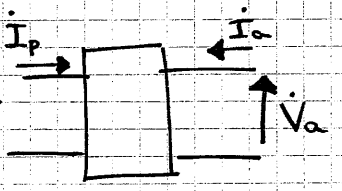
Equazioni di un doppio bipolo (obiettivo, studiare i componenti nel suo insieme.

$$\begin{bmatrix} \dot{V}_p \\ \dot{V}_a \end{bmatrix} = \begin{bmatrix} \bar{Z}_{pp} & \bar{Z}_{pa} \\ \bar{Z}_{ap} & \bar{Z}_{aa} \end{bmatrix} \cdot \begin{bmatrix} \dot{I}_p \\ \dot{I}_a \end{bmatrix}$$



impedenza a vuoto.

$$\begin{bmatrix} \dot{I}_p \\ \dot{I}_a \end{bmatrix} = \begin{bmatrix} \bar{Y}_{pp} & \bar{Y}_{pa} \\ \bar{Y}_{ap} & \bar{Y}_{aa} \end{bmatrix} \cdot \begin{bmatrix} \dot{V}_p \\ \dot{V}_a \end{bmatrix}$$

ammettenze di cortocircuito.

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costanti di trasmissione

$$\begin{bmatrix} \dot{V}_p \\ \dot{I}_p \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \dot{V}_a \\ \dot{I}_a \end{bmatrix}$$

→ Matrice di trasmissione

$$\begin{cases} \dot{V}_p = \bar{A}\dot{V}_a + \bar{B}\dot{I}_a \\ \dot{I}_p = \bar{C}\dot{V}_a + \bar{D}\dot{I}_a \end{cases}$$

$$\bar{A} = \frac{\dot{V}_p}{\dot{V}_a}\bigg|_{\dot{I}_a=0}$$

VUOTO $\dot{I}_a = 0$

prova a vuoto!!!

dalla stessa prova ricavo

$$\bar{C} = \frac{\dot{I}_p}{\dot{V}_a}\bigg|_{\dot{I}_a=0}$$

$$\bar{B} = \frac{\dot{V}_p}{\dot{I}_a}\bigg|_{\dot{V}_a=0}$$

PROVA DI CORTO

$$\bar{D} = \frac{\dot{I}_p}{\dot{I}_a}\bigg|_{\dot{V}_a=0}$$

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Passaggio da $|ABCD|$ a $|\bar{Z}|$ e $|\bar{Y}|$

$$\begin{cases} \dot{V}_p = \bar{A}\dot{V}_a + \bar{B}\dot{I}_a \\ \dot{I}_p = \bar{C}\dot{V}_a + \bar{D}\dot{I}_a \end{cases} \qquad \begin{cases} \dot{V}_p = \bar{Z}_{pp}\dot{I}_p + \bar{Z}_{pa}\dot{I}_a \\ \dot{V}_a = \bar{Z}_{ap}\dot{I}_p + \bar{Z}_{aa}\dot{I}_a \end{cases}$$

pongo $\dot{I}_a = 0$ (prova a vuoto)

$\dot{V}_p = \bar{A}\dot{V}_a$ $\qquad$ $\dot{V}_p = \bar{Z}_{pp}\dot{I}_p$

$\dot{I}_p = \bar{C}\dot{V}_a$ $\qquad$ $\dot{V}_a = \bar{Z}_{ap}\dot{I}_p$

$$\bar{A} = \frac{\dot{V}_p}{\dot{V}_a} = \frac{\bar{Z}_{pp}\dot{I}_p}{\bar{Z}_{ap}\dot{I}_p} \Rightarrow \boxed{\bar{A} = \frac{\bar{Z}_{pp}}{\bar{Z}_{ap}}}$$

$$\bar{C} = \frac{\dot{I}_p}{\dot{V}_a} = \frac{\dot{I}_p}{\bar{Z}_{ap}\dot{I}_p} = \frac{1}{\bar{Z}_{ap}} \Rightarrow \boxed{\bar{C} = \frac{1}{\bar{Z}_{ap}}}$$

pongo $\dot{V}_a = 0$ (prova di corto)

$\dot{V}_p = \bar{B}\dot{I}_a$ $\qquad$ $\dot{V}_p = \bar{Z}_{pp}\dot{I}_p + \bar{Z}_{pa}\dot{I}_a$

$\dot{I}_p = \bar{D}\dot{I}_a$ $\qquad$ $\dfrac{\dot{I}_p}{\dot{I}_a} = -\dfrac{\bar{Z}_{aa}}{\bar{Z}_{ap}}$

$$\bar{D} = \frac{\dot{I}_p}{\dot{I}_a} = -\frac{\bar{Z}_{aa}}{\bar{Z}_{ap}} \Rightarrow \boxed{D = -\frac{\bar{Z}_{aa}}{\bar{Z}_{ap}}}$$

$$\bar{B}\dot{I}_a = \bar{Z}_{pp}\bar{D}\dot{I}_a + \bar{Z}_{pa}\dot{I}_a$$

$$\bar{B} = \bar{Z}_{pp}\left(-\frac{\bar{Z}_{aa}}{\bar{Z}_{ap}}\right) + \bar{Z}_{pa} \Rightarrow \boxed{\bar{B} = \bar{Z}_{pa} - \bar{Z}_{pp}\left(\frac{\bar{Z}_{aa}}{\bar{Z}_{ap}}\right)}$$

$$\det \begin{vmatrix} A & B \\ C & D \end{vmatrix} = AD - BC = -\frac{\bar{Z}_{pa}}{\bar{Z}_{ap}} = -\frac{\bar{Y}_{ap}}{\bar{Y}_{pa}}$$

se il bipolo è reciproco $\bar{Z}_{pa} = \bar{Z}_{ap} \Rightarrow AD - BC = -1$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = a \qquad a^{-1} \rightarrow a^T = \begin{bmatrix} A & C \\ B & D \end{bmatrix}$$

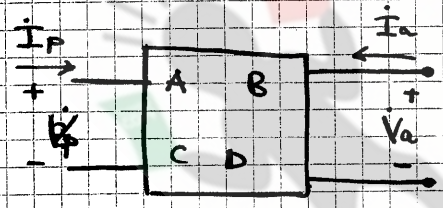
$$a^{-1} = \frac{1}{AD-BC} \cdot \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \rightarrow \text{se reciproco} \quad \begin{bmatrix} -D & B \\ C & -A \end{bmatrix} = a^{-1}$$

$$A = -D \qquad \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

Se $A = -D$ allora anche $\bar{Z}_{aa} = \bar{Z}_{pp}$

Il bipolo così è governato da soli 2 parametri.

IMPEDENZA ITERATIVA Z_{it}



$$\bar{Z}_p = \frac{\dot{V}_p}{\dot{I}_p}$$

$$\begin{cases} \dot{V}_p = \bar{A}\dot{V}_a + \bar{B}\dot{I}_a \\ \dot{V}_p = \bar{C}\dot{V}_a + \bar{D}\dot{I}_a \end{cases}$$

$$\dot{V}_a = \dot{V}_c \qquad \dot{I}_a = -\dot{I}_c$$

$$\dot{V}_c = \dot{I}_c \bar{Z}_c$$

$$\bar{Z}_p = \frac{\dot{V}_p}{\dot{I}_p} = \frac{\bar{A}\dot{I}_c\bar{Z}_c - \bar{B}\dot{I}_c}{C\dot{I}_c\bar{Z}_c - \bar{D}\dot{I}_c} = \frac{\bar{A}\bar{Z}_c - \bar{B}}{\bar{C}\bar{Z}_c - \bar{D}}$$

$$\bar{Z}_{it} = \frac{\bar{A}\bar{Z}_{it} - \bar{B}}{\bar{C}\bar{Z}_{it} - \bar{D}}$$

$$\bar{Z}_{it}^2 C - \bar{Z}_{it} D = \bar{A}\bar{Z}_{it} - B$$

$$Z_{it}^2 C - Z_{it}(A+D) + B = 0$$

$$Z_{it} = \frac{(A+D) \pm \sqrt{(A+D)^2 - 4CB}}{2C}$$

$$Z_{it} = \frac{(A+D) \pm \sqrt{(A+D)^2 - 4BC}}{2C}$$

se $A+D=0$ simmetrico

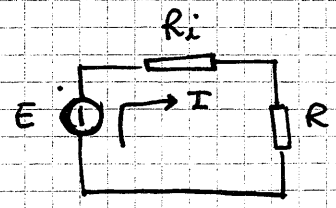
l'impedenza è caratteristica:

$$Z_c = \sqrt{-\frac{B}{C}}$$

Z_{it} = l'impedenza che collegata alla porta di ordine ridurre il bipolo ad un'impedenza della stessa valore.

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Trasferimento di potenza



$$P_R = I^2 R = \frac{R E^2}{(R_i+R)^2}$$

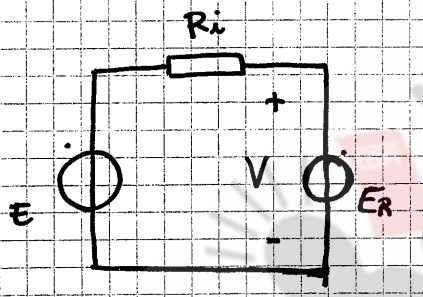
$$E = I(R_i+R)$$

P_{max} se $R_i = R$ $\qquad P_{max} = \frac{R E^2}{4R^2}$

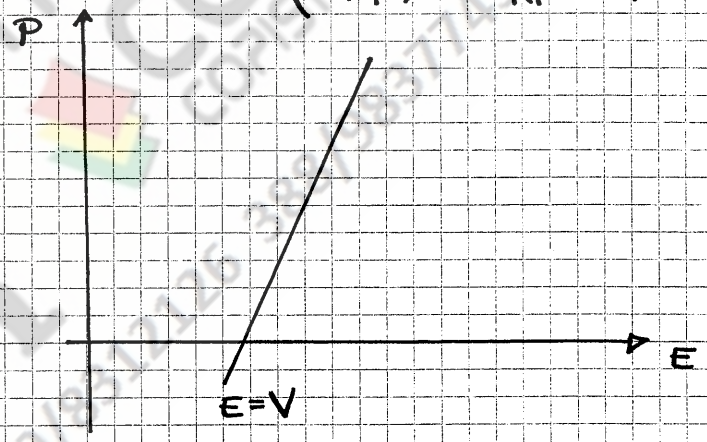
$$P_{max} = \frac{E^2}{4R}$$

η efficienza di trasferimento

$$\eta = \frac{RI^2}{EI} = \frac{R E^2}{(R_i+R)^2} \Big/ \frac{E^2}{(R_i+R)} = \frac{1}{1+\frac{R_i}{R}}$$



$$P = VI = V\left(\frac{E-V}{R_i}\right) = \frac{VE}{R_i} - \frac{V^2}{R_i}$$



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